Table of Notation

Symbol	Explanation	Page
$\mathrm{Atom}(U)$	set of atoms of the ideal U	84
$\mathrm{Aut}\, L$	automorphism group of L	12
B_n	boolean lattice with n atoms	4
C_n	n-element chain	4
$\mathrm{con}(a, b)$	smallest congruence under which $a \equiv b$	15
$\mathrm{con}(c)$	principal congruence for a color c	39
$\mathrm{con}(H)$	smallest congruence collapsing H	16
$\mathrm{con}(\mathfrak{p})$	principal congruence for the prime interval $\mathfrak{p}$	37
$\mathrm{Con}\, L$	congruence lattice of L	15, 48
$\mathrm{Con_J}\, L$	order of join-irreducible congruences of L	37
$\mathrm{Con_M}\, L$	order of meet-irreducible congruences of L	71
$\mathrm{Cube}\, K$	cubic extension of K	71
$\mathbf{D}$	class (variety) of distributive lattices	24
Diag	diagonal embedding of K into $\mathrm{Cube}\, K$	71
$\mathrm{Down}\, P$	order of down-sets of the (hemi)order P	4, 9, 232
$\mathrm{ext}\colon \mathrm{Con}\, K \to \mathrm{Con}\, L$	for $K \leq L$, extension map: $\Theta \mapsto \mathrm{con}_L(\Theta)$	41
$\mathrm{fil}(a)$	filter generated by the element a	14
$\mathrm{fil}(H)$	filter generated by the set H	14
$F_{\mathbf{D}}(3)$	free distributive lattice on three generators	26
$\mathrm{F}_{\mathbf{K}}(H)$	free lattice generated by H in a variety $\mathbf{K}$	26
$F_{\mathbf{M}}(3)$	free modular lattice on three generators	28
$\mathrm{Frucht}\, C$	Frucht lattice of a graph C	178
$\hom_{\{\vee,0\}}(X, Y)$	$\{\vee, 0\}$-homomorphism of X into Y	253

Symbol	Explanation	Page
$\mathrm{id}(a)$	ideal generated by the element a	14
$\mathrm{id}(H)$	ideal generated by the set H	14
$\mathrm{Id}\, L$	ideal lattice of L	14, 48
(Id)	condition to define ideals	14, 48
Isoform	class of isoform lattices	141
$\mathrm{J}(D)$	order of join-irreducible elements of D	19
$\mathrm{J}(\varphi)$	$\mathrm{J}(\varphi)\colon \mathrm{J}(E) \to \mathrm{J}(D)$, the "inverse" of $\varphi\colon D \to E$	32
$\mathrm{J}(a)$	set of join-irreducible elements below a	19
$\ker(\varphi)$	congruence kernel of φ	16
L	class (variety) of all lattices	25
M	class (variety) of modular lattices	25
Max	maximal elements of an order	49
$\mathbf{mcr}(n)$	minimal congruence representation function	87
$\mathbf{mcr}(n, \mathbf{V})$	**mcr** for a class **V**	87
$\mathrm{M}(D)$	order of meet-irreducible elements of D	32
M_3	five-element modular nondistributive lattice	xvii, 11, 30
$M_3[L]$	order of boolean triples of L	58
$M_3[L, a]$	interval of $M_3[L]$	63
$M_3[L, a, b]$	interval of $M_3[L]$	65
$M_3[a, b]$	order of boolean triples of the interval $[a, b]$	58
$M_3[\Theta]$	reflection of Θ^3 to $M_3[L]$	60
$M_3[\Theta, a]$	reflection of Θ^3 to $M_3[L, a]$	64
$M_3[\Theta, a, b]$	reflection of Θ^3 to $M_3[L, a, b]$	xvii, 67
N_5	five-element nonmodular lattice	xvii, 11, 30
$N_{5,5}$	seven-element nonmodular lattice	94
$N_6 = N(p, q)$	six-element nonmodular lattice	xvii, 80
$N_6[L]$	2/3-boolean triple construction	198
$N(A, B)$	lattice construction	132
$O(f)$	Landau O notation	xxvi
$\mathrm{Part}\, A$	partition lattice of A	7, 9
$\mathrm{Pow}\, X$	power set lattice of X	4
$\mathrm{Pow}^+ X$	order of nonempty subsets of X	219
$\mathrm{Prime}(L)$	set of prime intervals of L	37
$\mathrm{re}\colon \mathrm{Con}\, L \to \mathrm{Con}\, K$	reflection (restriction) map: $\Theta \mapsto \Theta \rceil K$	39
SecComp	class of sectionally complemented lattices	87
SemiMod	class of semimodular lattices	87
$\mathrm{Simp}\, K$	simple extension of K	71
$(\mathrm{SP}_\vee)$	join-substitution property	14, 48
$(\mathrm{SP}_\wedge)$	meet-substitution property	xvii, 14, 48
$\mathrm{sub}(H)$	sublattice generated by H	13
S_8	eight-element semimodular lattice	106
T	class (variety) of trivial lattices	25
Uniform	class of uniform lattices	141

Symbol	Explanation	Page
Relations and Congruences		
A^2	set of ordered pairs of A	3
ϱ, τ, π, ...	binary relations	
Θ, Ψ, ...	congruences	
ω	zero of Part A	7
ι	unit of Part A	7
$a \equiv b \ (\pi)$	a and b in the same block of π	7
$a \ \varrho \ b$	a and b in relation ϱ	3
$a \equiv b \ (\Theta)$	a and b in relation Θ	3
a/π	block containing a	6, 14
H/π	blocks represented by H	7
$\alpha \circ \beta$	product of α and β	21
$\alpha \overset{\mathrm{r}}{\circ} \beta$	reflexive product of α and β	30
$\Theta\rceil_K$	restriction of Θ to the sublattice K	14
L/Θ	quotient lattice	16
Φ/Θ	quotient congruence	16
π_i	projection map: $L_1 \times \cdots \times L_n \to L_i$	21
$\Theta \times \Phi$	direct product of congruences	21
Orders		
$\leq$, $<$	ordering	3
$\geq$, $>$	ordering, inverse notation	3
$K \leq L$	K a sublattice of L	13
$\leq_Q$	ordering of P restricted to a subset Q	4
$a \parallel b$	a incomparable with b	3
$a \prec b$	a is covered by b	5
$b \succ a$	b covers a	5
0	zero, least element of an order	4
1	unit, largest element of an order	4
$a \vee b$	join operation	9
$\bigvee H$	least upper bound of H	3
$a \wedge b$	meet operation	9
$\bigwedge H$	greatest lower bound of H	4
P^d	dual of the order (lattice) P	4, 10
$[a, b]$	interval	13
$\downarrow H$	down-set generated by H	4
$\downarrow a$	down-set generated by $\{a\}$	4
$P \cong Q$	order (lattice) P isomorphic to Q	4, 12

Symbol	Explanation	Page
Constructions		
$P \times Q$	direct product of P and Q	5, 20
$P + Q$	sum of P and Q	6
$P \dot{+} Q$	glued sum of P and Q	16
$A[B]$	tensor extension of A by B	248
$A \otimes B$	tensor product of A and B	245
$U \circledast V$	modular lattice construction	120
Perpectivities		
$[a,b] \sim [c,d]$	$[a,b]$ perspective to $[c,d]$	32
$[a,b] \overset{\text{u}}{\sim} [c,d]$	$[a,b]$ up-perspective to $[c,d]$	33
$[a,b] \overset{\text{d}}{\sim} [c,d]$	$[a,b]$ down-perspective to $[c,d]$	33
$[a,b] \approx [c,d]$	$[a,b]$ projective to $[c,d]$	33
$[a,b] \nearrow [c,d]$	$[a,b]$ up congruence-perspective onto $[c,d]$	35
$[a,b] \searrow [c,d]$	$[a,b]$ down congruence-perspective onto $[c,d]$	35
$[a,b] \hookrightarrow [c,d]$	$[a,b]$ congruence-perspective onto $[c,d]$	35
$[a,b] \Rightarrow [c,d]$	$[a,b]$ congruence-projective onto $[c,d]$	36
$[a,b] \Leftrightarrow [c,d]$	$[a,b] \Rightarrow [c,d]$ and $[c,d] \Rightarrow [a,b]$	36
Prime intervals		
$\mathfrak{p}$, $\mathfrak{q}$, ...		
$\operatorname{con}(\mathfrak{p})$	principal congruence generated by $\mathfrak{p}$	37
$\mathfrak{p} \Rightarrow \mathfrak{q}$	$\mathfrak{p}$ is congruence-projective onto $\mathfrak{q}$	36
$\mathfrak{p} \Leftrightarrow \mathfrak{q}$	$\mathfrak{p} \Rightarrow \mathfrak{q}$ and $\mathfrak{q} \Rightarrow \mathfrak{p}$	36
$\operatorname{Prime}(L)$	set of prime intervals of L	37
Miscellaneous		
$\overline{x}$	closure of x	10
$\varnothing$	empty set	4

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